Hidden Markov Models of Seismicity With Covariate Dependent Transition Probabilities

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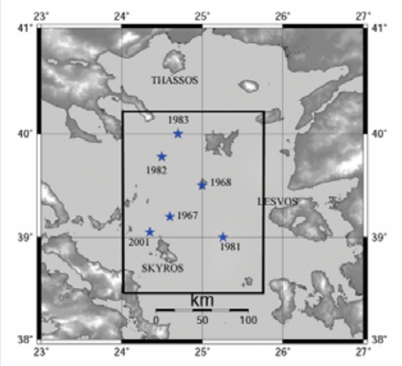
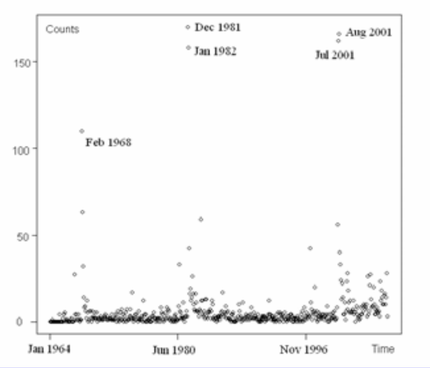
Seismic eArly warning For EuRope

Abstract: Earthquake counts observed in a particular region in successive time intervals tend to be dependent. Thus, appropriate statistical models, like the Hidden Markov Models (HMM) should be applied to incorporate such a dependent structure. In discrete valued Hidden Markov Models the observational sequence is the number of earthquakes observed in a fixed time unit for a pre-determined time interval. Furthermore, each observation is characterized by the unobserved state of seismicity during that time. The sequence of states is not independent. Each state depends on the previous one through a transition probability matrix and this creates dependence between the observed earthquake counts.

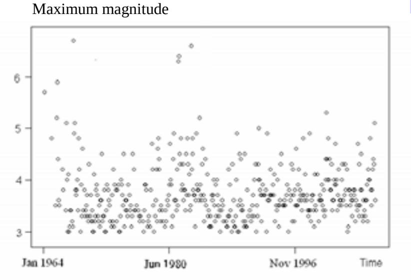
The present paper examines whether earthquake counts incorporate enough information for an observed value to be used to estimate the probability of moving from a particular state to a state of higher or lower seismicity. In addition, we examine whether the transition probabilities depend not only on the event counts but also on other factors. The mean and the maximum magnitude of the events are potential variables that may affect the transition probabilities. We incorporate these variables into the model as covariates that affect the transition probabilities through a logit link function and examine how the later changes over time. The models have been applied in an area of high seismicity in the northwestern part of the Aegean Sea (Greece) with the aim to examine the relationships between the covariate information and the transition probabilities.





Data: The North Aegean Sea seismic zone corresponds to the area represented by the rectangle at the left-hand side map. The blue stars on the map show the epicenters of the strongest (M≥6.5) earthquakes that occurred in the region the last 35 years. The data have been selected from the catalogue of the Institute of Geodynamics, National Observatory of Athens (http://www.gein.noa.gr/) for the time interval from 1964 up to 2007 inclusive. Completeness analysis based on the magnitude-frequency relationship showed that the data are complete for local magnitudes $M_l \geq 3.0$. The observational sequence consists of 528 successive counts. Each count corresponds to the number of seismic events that occurred in every particular month.

Maximum magnitude



Covariate	Log Likelihood
No covariate	-2123.511
Maximum magnitude	-2097.759
Previous maximum magnitude	-2120.546
Mean magnitude	-2120.767

Covariates: We include 3 different sets of variables into the model. These are: 1) the maximum earthquake magnitude observed in each count (the temporal distribution of the maximum magnitude over time is illustrated on the left - hand side figure), 2) the mean magnitude observed in each count and 3) the maximum magnitude observed in the previous count.

Markov Poisson Regression Models:Definition and Notation

- MPRM are discrete time stochastic processes that consist of two parts: an observed sequence $\{(S_t, x_t): t \in \mathbb{N}\}$, where S_t is a non-negative integer valued stochastic process and $\{x_t : x_t = (x_{t1,\dots,} x_{tp}), t \in \mathbb{N}\}$ a vector of p covariates and an unobserved finite state Markov chain on m states $\{C_t : t \in \mathbb{N}\}$. For all positive integers T, conditionally on $C^{(T)} = C_t : t = 1, \dots, T\}$ the random variables $S_1, \dots, S_T$ are independent.
- The marginal distribution of S_t is:

$$p(s_t) = \sum_{j=1}^{m} a_j f(s_t \mid \lambda_j), \quad where \quad a_i > 0, \; i = 1, \dots, m, \quad \sum_{i=1}^{m} a_i = 1,$$

and

$$f(s \mid \lambda) = \frac{e^{-\lambda} \lambda^s}{s!}, s = 0, 1, \dots, \quad \lambda \geq 0$$

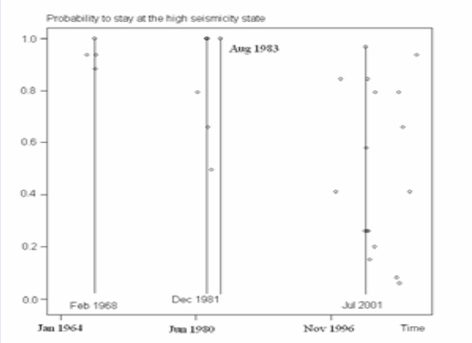
- The conditional distribution of S_t given $C^{(T)}$ is:

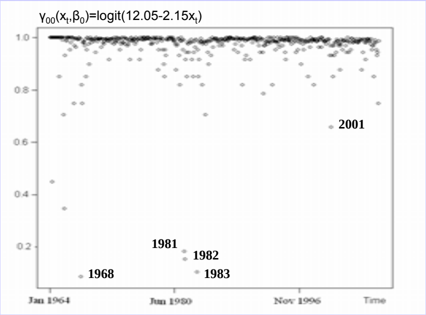
$$\pi_{s_t i} = P(S_t = s_t \mid C_t = i) = \frac{e^{-\lambda_i} \lambda_i^{s_t}}{s_t!}, \quad s = 0, 1, \dots, \quad \lambda \geq 0$$

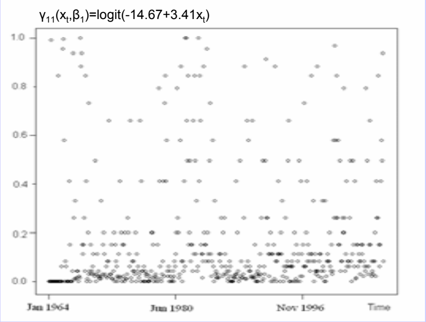
- The transition probabilities of the Markov chain $\gamma_{ij} = P(C_t = j \mid C_{t-1} = i)$ are associated with covariates through a logit link function:

$$P(C_t = 0 \mid C_{t-1} = 0) = \gamma_{00}(x_t, \beta_0) \equiv \operatorname{log}it(\beta_0' x_t)$$
$$P(C_t = 1 \mid C_{t-1} = 0) = \gamma_{01}(x_t, \beta_0) \equiv 1 - \gamma_{01}(x_t, \beta_0)$$
$$P(C_t = 1 \mid C_{t-1} = 1) = \gamma_{11}(x_t, \beta_1) \equiv \operatorname{log}it(\beta_1' x_t)$$
$$P(C_t = 0 \mid C_{t-1} = 1) = \gamma_{10}(x_t, \beta_0) \equiv 1 - \gamma_{11}(x_t, \beta_1)$$

Where $\beta_0 = (\beta_{00}, \beta_{01}, \dots, \beta_{0p}), \beta_1 = (\beta_{10}, \beta_{11}, \dots, \beta_{1p})$ are unknown parameters to be estimated







Results

1. The models with covariates improve the model without covariates. In particular , in the case of the model with the maximum magnitude the improvement is significant at a significance level of 1% based on the Likelihood Ratio Statistic. The maximum magnitude is the variable that has the most significant effect on the transition probabilities. So, all the results presented concern the particular variable.
2. The two constant rate values capture the extreme values of seismicity rate: the low one (with rate λ_0=3.61 number of events/month) and a high one (with rate λ_1=55.78 number of events/month)
3. If two counts are both associated with states of **low** seismicity during a time period, given that β_{01}=-2.15 is significantly negative, then the count with the lower maximum magnitude will be more likely to remain in the low state of seismicity during the next time interval, while the other with the higher maximum magnitude will be more likely to transit to the high state of seismicity.
4. If two counts are both associated with states of **high** seismicity during a time period, given that β_{11}=3.41 is significantly positive, the count with the lower maximum magnitude will be more likely to transit to a low state of seismicity during the next time interval, while the other with the higher maximum magnitude will be more likely to remain in the high state of seismicity.
5. The model describes adequately the data. In the months that correspond to the 1968, 1981, 1982, 1983 and 2001 strong earthquakes the probability to remain at the stay of high seismicity is very close to 1.